Hygrothermally Induced Postbuckling Response of Non-linear Elastically Supported Laminated Composite Plates with Uncertain System Properties

Stochastic Finite Element Macromechanical Model

Rajesh Kumar*[1], H S Patil[2]

[1] Deptt. of Mechanical Engineering, Hindu College of Engineering, Sonepat-131001 (HR), India

[2]Deptt. of Applied Mechanics, S.V.National Institute of Technology, Surat-395007, India

[1]rajeshtripathi63@gmail.com; patilhs57@gmail.com

Abstract

In the present study, post buckling response of hygrothermally induced non-linear elastically supported laminated composite plate with uncertain system properties using stochastic finite element macromechanical model approach is investigated. Uncertain system properties such as thermal expansion coefficients, hygroscopic expansion coefficients, foundation stiffness parameters, geometric properties and lamina material properties are modeled as basic random variables. The basic formulation is based on a higher-order shear deformation plate theory in von Karman nonlinear kinematics. A direct iterative based C^0 non linear finite in conjunction with Taylor series based mean-centered first order regular perturbation technique be used to obtain mean and standard deviation of post buckling load of laminated composite plates. The effects of rising temperature, degree of moisture concentrations, boundary conditions, plate aspect ratios, total number of plies, nonlinear elastic foundation stiffness parameters and fiber orientations are analysed. The approach has been validated by comparison of results with availability in literature and independent Monte Carlo simulation.

Keywords

Hygrothermally Induced Post Buckling; Uncertain System Properties; Nonlinear Elastic Foundations; Laminated Composite Plates; Regular Perturbation Technique

Introduction

Nonlinear elastically supported laminated composite plates have been used in a great variety of engineering applications including aerospace vehicles, aircraft runways, launching sites of missile and spacecraft, foundation engineering etc. The applications have been increased in many folds for the aerospace and other engineering fields due to combined features of high strength to weight ratio, excellent corrosion resistance and good fatigue characteristics. They operate in a variety of elevated thermal and moisture environments that may have a pronounced impact on their performance during their service life. These hygrothermal effects are the result of the temperature and moisture content variations and are related to the difference in the thermal and hygro properties of the constituents. The varying environmental conditions due to moisture absorption and temperature have an adverse effect on the stiffness and strength of the composites. As, the matrix is more susceptible to the hygrothermal condition than the fiber, the deformation is observed to be more in the transverse direction of the composite. The rise in moisture and temperature reduces the elastic moduli and degrade the strength of the materials as well as induces internal initial stresses, which may affect the stability as well as the safety of the structures.

As a result, a careful evaluation of environmental exposure is required to find the nature and extent of their adverse effects upon performance design in composite analysis and uncertainties in system behavior. Macro-mechanical stochastic modeling yields more accurate and exact prediction of system behavior and has proved to be superior design. Large numbers of parameters are associated with manufacturing and fabrication composite components as compared to conventional material. For examples material and geometric properties, fiber orientations, lamina lay-up sequence design and curing parameters. Effects of these parameters must be considered in accurate prediction of system behavior of composite so

that reliability of the structure during it operating life can be assessed accurately. Thus there is a need to quantify structural system uncertainties in the response. This may be appropriately handled by modeling system properties in probabilistic sense only. Uncertainties in the system properties lead to uncertainties in the response behavior of the structure. For accurate study of structural behavior, the random variation in the system properties should be incorporated in the analysis. Otherwise the predicted response may differ significantly from the observed values and the structures may not be safe.

In the existing literature of the published works based on deterministic analysis using macro mechanical model on hygrothermal buckling analysis, notably published work of Whitney and Ashton [1971], Flaggs et al. [1978], Lee and Yen [1992], Ram and Sinha [1992], hygrothermal buckling analysis using micro-mechanical model are based on deterministic analysis Shen [2001]. Hygrothermal effects on the structural behavior of thick laminates applying higher order theory of a deterministic analysis has been studied by Patel et al. [2002]. In deterministic analysis the system parameters are taken to be deterministic and variation in the system parameters are ignored which result in a conservative design and decrease the potential of the composites. It is therefore necessary that the system properties are taken as random variables. Limited work has been done assuming system properties as random and their effects on the performance of composites. Nakagiri et al. [1990] studied all edges simply supported of a laminate with stochastic finite element method taking fiber orientation, layer thickness and number of layers as random variables. Englested and Reddy [1994] contributed metal matrix composites based on probabilistic micro mechanics non-linear analysis. Singh at al. [2001] studied the buckling of composite cylindrical panels with uncertain material properties using probabilistic approach. Effects of random system properties on initial buckling of composite plates resting on elastic foundation using stochastic finite element method was studied by Lal et al. [2008]. Post buckling of laminated composite plate on elastic foundation with random system properties based on HSDT in conjunction with FOPT was studied by Singh et al. [2009]. Chen at al. [1992] outlined the probabilistic method to evaluate the effects of uncertainties in geometric and material properties. Relatively little work is available on hygrothermal linear buckling analysis of the structures made of composites with random system parameters using macro-mechanical model Singh and Verma [2009]. Pandey et al. [2009] studied the thermo elastic stability analysis of laminated composite plates resting on nonlinear elastic foundations using analytical approach. Singh et al. [2001, 2001] studied the effects of random system properties on initial buckling and thermal buckling using stochastic finite element method. Onkar et al. [2006, 2007] studied the buckling problem using stochastic finite element problems. Lal et al. [2009] studied the effects on the thermal buckling response of laminated composite plates using stochastic finite element method, Verma et al.[2009] investigated thermal buckling problem with random material and geometric properties.

However, no work dealing with hygrothermal post buckling analysis of the laminated composite plate with random system properties i.e. material properties, geometric properties and nonlinear elastic foundations parameters using macro mechanical model is reported in the literature to the best of the authors' knowledge. The present study is a further extended work of Singh et al. [2009] contributions to provide a tool for analysis of structural system uncertainties in elevated hygrothermal environments especially applicable for aerospace engineering and other fields.

Formulations

A rectangular arbitrary laminated composite plate with of length a, width b, and total thickness h, being defined in (X, Y, Z) system with X- and -Y axes located in the middle plane and its origin placed at the corner of the plate. Let $(\bar{u}, \bar{v}, \bar{w})$ be the displacement paralleling to the (X, Y, Z) respectively as shown in Figure 1. The thickness coordinate Z of the top and bottom surfaces of any k_{th} layer are denoted by $Z_{(k-1)}$ and $Z_{(k)}$ respectively. The fiber of the k_{th} layer is oriented with angle θ_k to the X- axes. The plate is assumed to be attached to the foundation so that no separation takes place in the process of deformation. The load – displacement relation between the plate and the supporting foundations is given as

$$P=K_1 w+K_3 w^3-K_2 \nabla^2 w \tag{1}$$

where P is the foundation reaction per unit area, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ and K_1, K_2 and K_3 are linear Winkler (normal) foundation, linear Pasternak (shear layer) foundation and nonlinear Winkler foundation parameters, respectively and w is the transverse displacement of the plate. This model is simply known as Winkler type when $K_2 = 0$.

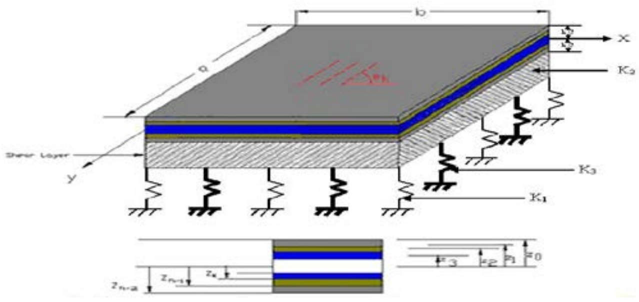

FIG.1 GEOMETRY OF LAMINATED COMPOSITE PLATE WITH NONLINEAR ELASTIC FOUNDATION

Displacement field , Strain Displacement Relations, Stress-Strain Relation ,Energy Equations The Strain Energy due to Nonlinear Elastic Foundation, Potential Energy due to Hygrothermal Stresses, Finite Element Formulations are studied in Singh[2009], Reddy[1984], Reddy[1997], John[1975], Agrawal[1990], Singh[2002], Shankara[1996] and Franklin[1968] .

The Equation of Motion and its Solution Technique

Governing Equation, Steps for the Direct Iterative Technique and solution Technique are given in Reddy [1981], Kleiber [1992], Graham [2001], Yamin [1996], Liu [1986] and Graham [2001].

Results and Discussions

The direct iterative based stochastic finite element method [DISFEM] is used to illustrate the hygrothermally induced post buckling response considering different random system parameters of laminated composite plate i.e. symmetric and antisymmetric.

In this section the mean post buckling load and effect of randomness in the system properties on the postbuckling load of geometrically non linear laminated composite plate resting on nonlinear elastic foundations are investigated in hygrothermal environments. The present methodology for mean and coefficient of variation of post buckling load in hygrothermal environment is verified with available results in literature and independent MCS. The lamina coefficients of hygroscopic expansion, thermal expansion and foundation stiffness parameters including geometric and material properties are modeled as basic random variables (RVs) as stated earlier. However, the results are only presented taking coefficient of variation (COV) of the system property equal to 0.10 Liu [1986] as the nature of the SD (standard deviation) variation is linear and passes through the origin. Hence, the presented results would be sufficient to extrapolate the results for other COV value keeping in mind the limitation of the FOPT Singh et al.[2009,2002]. The thickness of all the lamina is assumed to be constant and of same material without varying individual property of materials used. The obtained results have been compared with MCS and those available in the literature. For the present study a nine nodded serendipity element with 63 degree of freedom (DOFs) per element, a modified form HSDT based C^0 finite element model has been used for discretizing the laminate. Based on convergence, an (8x8) mesh has been used throughout for evaluation of the results. Programme in Mat lab is developed for the computational analysis. Results have been computed by employing the full (3x3) integration rule for bending stiffness matrices, hygrothermal load vector and the geometric stiffness matrices and the reduced (2x2) integration rule for computing the shear stiffness matrices to avoid shear locking in the thin plates.

All edges simply supported (SSSS (S1)):

$$u = w = \theta_y = \psi_y = 0,\ at\ x = 0, a; \quad v = w = \theta_x = \psi_x = 0\ at\ y = 0, b\ ;$$

All edges simply supported (SSSS (S2)):

$$v = w = \theta_y = \psi_y = 0,\ at\ x = 0, a; \quad u = w = \theta_x = \psi_x = 0\ at\ y = 0, b\ ;$$

All edges clamped (CCCC):

$$u = v = w = \psi_x = \psi_y = \theta_x = \theta_y = 0,\ at\ x = 0, a \quad and\ \ y = 0, b;$$

The used plate geometry is characterized by aspect ratios (a/b) = 1and 2, side to thickness ratios (a/h) = 10, 20, 30, 40, 50, 60, 80 and 100. The mean values of coefficients of hygroscopic expansion, coefficients of thermal expansion, geometric properties, material constants and foundation stiffness parameters are used for computation. We consider now a second steps as the elastic constants, thermal expansion coefficients and coefficients of hygroscopic expansion of each layers are assumed to be linear function of temperature and moisture. The only exception is the Poisson's ratio, which can be assumed as constant due to weakly dependency on temperature and moisture change. The coefficient of hygroscopic expansion β_{11}in longitudinal direction is considered to be zero because moisture contents have negligible effects on fiber and have effects on matrix in transverse direction. For the coefficient of variation of material and geometric properties varying from 0 to 20% with a/h= 10, 20, 30, 40, 50, 80 and 100 subjected to uniform temperature and moisture rise have been considered. The lamina material properties such as E_{11}, E_{22}, G_{12}, G_{13}, G_{23}, α_1, α_2,

β_2 and h are modeled as basic RVs. E_{11}, and E_{22} are longitudinal and transverse elastic modulii, respectively, G_{12}, G_{13}, G_{23} are shear modulii, α_1 and α_2 are longitudinal and transverse coefficients of thermal expansion, β_1 and β_2 are longitudinal and transverse coefficients of hygroscopic expansion and h are the total thickness of the composite laminate considered for the analysis. The input random variables b_i is related and sequenced as

$b_1= E_{11}$, $b_2=E_{22}$, $b_3=G_{12}$, $b_4= G_{13}$, $b_5=G_{23}$, $b_6=\alpha_1$, $b_7=\alpha_2$, $b_8= \beta_2$ and $b_9=h$. The input random variables for plate resting on nonlinear elastic foundations are related as $b_1= E_{11}$, $b_2=E_{22}$, $b_3=G_{12}$, $b_4= G_{13}$, $b_5=G_{23}$, $b_6=\alpha_1$, $b_7=\alpha_2$, $b_8= \beta_2$, $b_9=k_1$, $b_{10}=k_2$ and $b_{11}=k_3$. The non dimensionalised foundation parameters used for the analysis are as

$$k_1 = K_3 a^4 / E_{22}^d h^3 \quad ; \quad k_2 = K_2 a^4 / E_{22}^d h^3 \quad ; \quad k_3 = K_3 a^4 / E_{22}^d h^2 \, .$$

Table 1(*a*), 1(*b*) and 1 (*c*) are compared for validation of mean post buckling analysis. Tables 2(*a*), 3(*a*), 4(*a*), 5(*a*) and 7(*a*) give generated results for various combinations of random variables with geometric parameters for second order statistics for post buckling load of laminated composite plate in hygrothermal environment. The numerical values and relationship between the mean values of the material properties for graphite epoxy composite with change in temperature and moisture.

For plates resting on nonlinear elastic foundations Tables 1(*d*), 1(*e*) and 1(*f*) are compared for validation of mean post buckling analysis. Tables 2(*b*), 3(*b*), 4(*b*), 5(*b*), 5(*c*), 6(*a*), 6(i), 7(*b*) and 7(*c*) give generated results for various combinations of random variables with geometric parameters for second order statistics for post buckling load of laminated composite plate resting on nonlinear elastic foundations in hygrothermal environment. The numerical values and relationship between the mean values of the material properties for graphite epoxy composite with change in temperature and moisture.

Validation Study: Mean and Standard Deviations Without Foundations

In order to validate the proposed outlined approach, the results for the mean and standard deviation are compared with those available in the literature and an independent Monte Carlo simulation technique.

1) Mean Hygrothermal Post Buckling Analysis

Table 1 (a) & Table 1(b) show the comparison of effect of temperature and moisture respectively on the critical load $\lambda_{crl}= N_{xcr}/ (N_{xcr})_{c=0\% \text{ or } T=300K}$ of anti-symmetric cross ply $(0^0/90^0)_{2T}$, clamp supported laminate, side to thickness ratio *a*/*h* =10 with those available results. The results are in good agreement. Table 1(c) shows the comparison of effect of moisture and temperature on the dimensionalised buckling load of cross ply $[0^0/90^0]$, clamped supported square laminate with those available results of Singh and Verma.[2009]. The present results are in close tolerance with the available literature. Table 1 (d), 1(e) show the comparison of effect of temperature and moisture respectively on the critical load $\lambda_{crl}= N_{xcr}/ (N_{xcr})_{c=0\% \text{ or } T=300K}$ of anti-symmetric cross ply $(0^0/90^0)_{2T}$, simply supported S2 laminate, plate thickness ratio (*a*/*h*) =10 with those available results SaiRam and Sinha [1992].The results are in good agreement. Table 1(f) shows the comparison of parametric studies results for aspect ratio a/b=1, thickness ratios a/h=10, simple support SSSS (S2), biaxial compression , angle ply $[\pm 45^0]_{2T}$, on dimensionless mean thermal post buckling load of laminated composite plate subjected to temperature and moisture independent (TID) material properties. It is observed that foundation parameters have significant effects as shown by bounds of buckling load and post buckling strength. The material properties of the composite plate are: E_1/E_2 =25; $G_{12}=0.5E_2$; $G_{23}=0.2E_2$; E_{20}=1*1e5; v_{12}=0.25; E_{10}=m1*E_{20}; G_{120}=m2*E_{20}; $G_{130}=G_{120}$; G_{230}=m3*E_{20}; $v_{21}=v_{12}*E_{20}/E_{10}$; α_{110} =1*1e-6; α_{210} =10.0*1e-6; α_{12} = 0 ; αo =1e-6; The non dimensionalised foundation parameters are as; $K_1= k_1*D_{11} / a^4$, $K_2= k_2*D_{11} / a^2$, $K_3= k_3*D_{11} / a^4*h^2$ and ($T_{cr} = T_1* \alpha_0$*1000). The results obtained are in good agreements and validated with the available literature results of Pandey et al. [2009].

2) Validation Result for Random Material Properties

As mentioned earlier, no results are available in reported literature for variance of post buckling load in hygrothermal environment using HSDT based DISFEM. The verification of the results for COV of hygrothermal post buckling load is compared with available results obtained by MCS approach using samples of random number generation. The present direct iterative method in conjunction with first order perturbation technique [DISFEM] results for random response has been validated with an independent MCS approach.

TABLE 1 (A) &TABLE 1 (B)

a/b		Temperature T(K)					
		300	325	350	375	400	425
0.5	SaiRam and Sinha [1992]	1.000	0.988	0.951	0.911	0.888	0.865
	Present	1.000	0.991	0.954	0.913	0.890	0.868
2.0	SaiRam and Sinha [1992]	1.000	0.994	0.931	0.865	0.828	0.792
	Present	1.000	0.995	0.930	0.862	0.827	0.792

a/b		Moisture Concentration C(%)						
		0.00	0.25	0.50	0.75	1.00	1.25	1.50
0.5	SaiRam and Sinha [1992]	1.000	0.992	0.984	0.977	0.970	0.965	0.960
	Present	1.000	0.9916	0.9833	0.9752	0.9672	0.960	0.953
2.0	SaiRam and Sinha [1992]	1.000	0.997	0.993	0.989	0.987	0.985	0.982
	Present	1.000	0.997	0.994	0.991	0.989	0.987	0.985

TABLE 1(C)

a/h	Singh and Verma [2009]			Present [HSDT]	
	Moisture concentration *C* (%)	Temperature *T* (K)	buckling load N_x (N/m) (10^8)	E_2 & G_{12}	buckling load N_x (N/m) x (10^8)
10	0.1	300	7.475	9.5 /6.0	7.4412
20	0.5	350	1.1536	9.0 / 6.0	1.5837
50	1.0	400	0.097	8.5/ 6.0	0.1102

TABLE 1 (D)

a/b		Temperature T(K)					
		300	325	350	375	400	425
0.5	SaiRam and Sinha [1992]	1.000	0.983	0.955	0.926	0.907	0.889
	Present	1.000	0.986	0.963	0.939	0.923	0.910

TABLE 1 (E)

a/b		Moisture Concentration C(%)						
		0.00	0.25	0.50	0.75	1.00	1.25	1.50
0.5	SaiRam and Sinha [1992]	1.000	0.982	0.965	0.949	0.934	0.920	0.906
	Present	1.000	0.984	0.968	0.953	0.939	0.926	0.912

TABLE 1(F)

(W_{max}/h)	Pandey et al.[2009]	Present[HSDT]
0.0	9.850	9.981
0.2	10.076	10.157
0.4	10.998	11.550
0.6	12.282	11.898

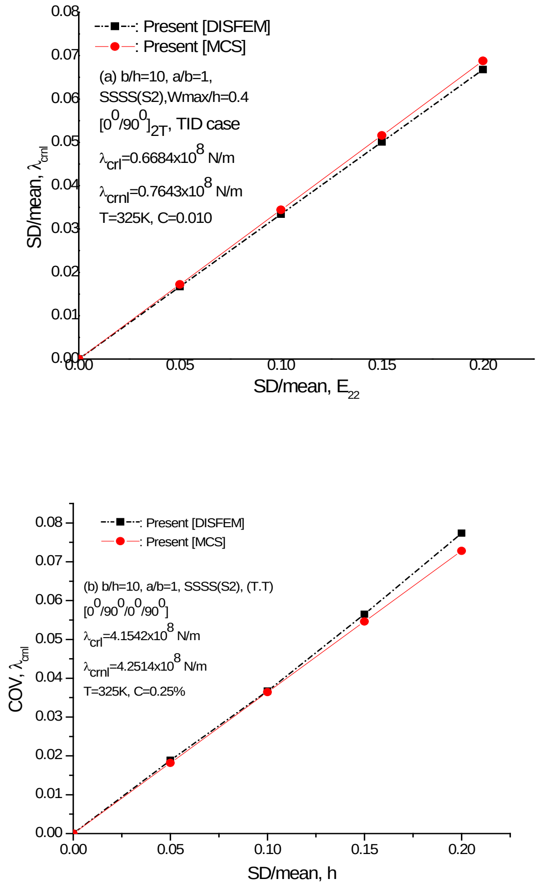

Figure. 2 Validation of present DISFEM results with independent MCS results for: (a) Random material properties (b) Random geometric properties.

Fig. 2 (a) plots the normalized standard deviation, coefficient of variation of hygrothermal post buckling load versus the coefficient of variation of the random material constants of cross-ply $[0^0/90^0/0^0/90^0]$ square laminated composite plates, SSSS (S2), a/h=20 with amplitude ratio (W_{max}/h =0.2) subjected to combination of uniform temperature and transverse temperature distribution changing from 0 to 20%. It is assumed that one of the material properties (i.e., E_{11}) changes at a time keeping other as a deterministic, with their mean values of the material properties. The dashed line is the present [DISFEM] result and the solid line is independent MCS approach. For the MCS approach, the samples are generated using Mat Lab to fit the mean and standard deviation. These samples are used in response equation Eq. (25), which is solved repeatedly, adopting conventional eigenvalue procedure, to generate a sample of the hygrothermal post bucking load. The number of samples used for MCS approach is 10,000 based on satisfactory convergence of the results. The normal distribution has been assumed for random number generations in MCS. It can also be observed that the present DISFEM results are in good agreement with MCS result.

3) Validation for Random Geometric Properties

Figure 2(b) demonstrates a comparison of the obtained result in the present DISFEM and independent MCS approach treating random lamina thickness as random variables keeping others as constant. The composite laminate having stacking sequence $[0^o/90^0]_{2T}$ thickness ratio a/h= 10 and amplitude ratio W_{max}/h=0.2 are used in the analysis. For the range of COV in the geometric properties such as lamina plate thickness (h) changing at a time keeping other as deterministic i.e. mean values are considered i.e. 0 to 20%. It is concluded that DISFEM as shown by dotted line gives acceptable results with the solid line of independent MCS using 12,000 samples of random number generation for the range of dispersion taken in the study.

Parametric Analysis of Second Order Statistics Without Foundations

Table 2 shows the effects of amplitude ratios (W_{max}/h) and nonlinear elastic foundation parameters with individual random system property b_i, {(i =1 to 11) = 0.10} on the dimensionlised mean (T^*_{crnl} = T_{crnl} e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load of cross ply $[0^0/90^0]_{2T}$ square laminated composite plates, subjected to uniform constant temperature and moisture (U.T) and in-plane uni-axial compression, plate thickness ratio (a/h) =20, rise in temperature and moisture ΔT= 325K, ΔC= 0.010, with simple support S2 boundary conditions. (T^*_{crl} = T_{crl}e+08) - mean dimensionalized linear solution. It is seen that the effects of amplitude ratios increases the mean values and COV for plate resting on nonlinear Winkler foundation. These values further increase when the plate is resting on Pasternak nonlinear elastic foundation. The mean values and COV are higher for input random variable E_{11} for plate resting on (k_1=100, k_2=10, k_3=200) Pasternak nonlinear elastic foundation. However the effects on coefficient of thermal expansion α_{11} are fever compared to all other input random variables .

Table 3(a) shows the effects of plate thickness ratios (a/h) , amplitude ratios (W_{max}/h) and the random input

variables b_i, {i =(1..8), (6, 7), (8) and (9) = 0.10}.on the dimensionlised mean ($T^*_{crnl} = T_{crnl}e+08$) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load of cross ply $[0^0/90^0]_{2T}$ square laminated composite plates with simple support (S1) and clamped support boundary conditions, rise in temperature and moisture (ΔT=325K, ΔC=0.25%). It is seen that for simple support S1 conditions, the mean hygrothermal post buckling load decreases as the side to thickness ratio increases while on increasing the amplitude ratio the result is reversed. The effects of combined random system property except lamina thickness with increase side to thickness ratio and amplitude ratio the coefficient of variation of hygrothermal post buckling load of the plate increases. In the case of plate with clamped support conditions, the mean value with different combinations increase much more compared to plate with simple support conditions. However, the effects on COV with different random system properties vary significantly in case of plate with clamped support conditions keeping same temperature and moisture conditions.

Table 3(b) shows the effects of plate thickness ratios (a/h), nonlinear elastic foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1.. . 11), (6, 7),(8) and (9..11)} = 0.10] on the dimensionalised mean ($T^*_{crnl} = T_{crnl}$ e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load of cross ply $[0^0/90^0]_{2T}$, square laminated composite plate, subjected to uniform constant temperature and moisture , in-plane uni-axial compression with simple support S2 boundary conditions. ΔT= 325K, ΔC= 0.010, ($T^*_{crl} = T_{crl}e+08$) N/m – mean dimensionalised linear solution in brackets. It is observed that for a given plate thickness ratio the effects of increase in amplitude ratio increases the mean values for the plate resting on nonlinear elastic foundations and these values further increase for Pasternak nonlinear elastic foundations. The effects on COV are of significance for all combined random input variables. If the plate thickness ratio is further increased then mean values further decreases and COV is significant for all combined random input variables and Winkler nonlinear elastic foundations compared to Pasternak nonlinear elastic foundations.

Table 4 shows the effects of aspect ratios (a/b) , nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10 on the mean ($T^*_{crnl} = T_{crnl}$ e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load of angle ply $[\pm45^0]_{2T}$ laminated composite plate, subjected to uniform constant temperature and moisture, in-plane uni-axial compression with a/h=30. ΔT= 325K, ΔC= 0.010. ($T^*_{crl} = T_{crl}e+08$) - mean linear solution. It is noticed that for square plate the effects of amplitude ratios increases the mean values for plate resting on Winkler nonlinear elastic foundations and these values are further higher for plate resting on Pasternak nonlinear elastic foundations, however there is appreciable increase in coefficient of variations. For rectangular plate the mean values sharply decline and COV increase significantly and is dominant for all combined input random variables. The COV is also higher for β_{22} when plate rests on Winkler nonlinear elastic foundations .

Table 5(a). shows the effects of support conditions, nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10] on the dimensionlised mean ($T^*_{crnl} = T_{crnl}$ e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load, ,plate thickness ratio (a/h) =20, of angle ply $[\pm45^0]_{2T}$ laminated square composite plate, subjected to of uniform constant temperature and moisture, in-plane uni-axial compression, ΔT= 325K, ΔC= 0.010, ($T^*_{crl} = T_{crl}e+08$) - mean linear solution. It is seen that for simple support SSSS (S1) square composite plate resting on Winkler nonlinear elastic foundation the increase in amplitude ratio increases the mean values and COV decreases. Mean values are further higher for plate resting on Pasternak nonlinear elastic foundations. The combined effects of all input random variables are dominant for Pasternak nonlinear elastic foundations. When the plate is simply supported SSSS (S2) and resting on Winkler or Pasternak nonlinear elastic foundations there is marginal variation in mean values and COV.

Table 5(b) shows the effects of support conditions, nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10] on the mean ($T^*_{crnl} = T_{crnl}$ e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load, ,plate thickness ratio (a/h) =20, of angle ply $[\pm45^0]_{2T}$ laminated square composite plate, subjected to of uniform constant temperature and moisture, in-plane uni-axial compression, ΔT= 325K, ΔC= 0.010, ($T^*_{crl} = T_{crl}e+08$) - mean linear solution. It is seen that for clamped support CCCC square composite plate resting on Winkler nonlinear elastic foundation the increase in amplitude ratio significantly increase the mean values and COV decreases. Mean values are further higher for

TABLE 2

b_i	W_{max}/h	COV, λ_{crnl}		
		(k_1=100, k_2=00 ,k_3=100)	(k_1=100, k_2=10, k_3=100)	(k_1=100, k_2=10, k_3=200)
E_{11} (i=1)	0.2	(T_{crnl} = 0.1887) 0.1053	(T_{crnl} =0.3324) 0.1523	(T_{crnl} =0.3345) 0.1550
	0.4	(T_{crnl} = 0.2057) 0.1023	(T_{crnl} =0.3803) 0.1482	(T_{crnl} =0.3881)0.7227
	0.6	(T_{crnl} = 0.2290) 0.0965	(T_{crnl} =0.4128)0.2339	(T_{crnl} =0.4947)0.2274
	T_{crl}	(0.1826)	(0.3003)	(0.3003)
E_{22} (i=2)	0.2	0.0677	0.0589	0.0586
	0.4	0.0644	0.0594	0.0581
	0.6	0.0633	0.0533	0.0579
G_{12}(i=3)	0.2	0.0098	0.0052	0.0052
	0.4	0.0091	0.0050	0.0049
	0.6	0.0081	0.0062	0.0057
G_{13} (i=4)	0.2	0.0023	0.0068	0.0068
	0.4	0.0024	0.0077	0.0076
	0.6	0.0036	0.0048	0.0100
G_{23} (i=5)	0.2	0.0012	0.0034	0.0034
	0.4	0.0012	0.0038	0.0038
	0.6	0.0018	0.0024	0.0050
	0.2	0.0014	7.98e-04	7.93e-04
α_{11} (i=6)	0.4	0.0013	6.97e-04	6.83e-04
	0.6	0.0012	6.42e-04	5.36e-04
α_{22} (i=7)	0.2	0.0110	0.0062	0.0062
	0.4	0.0101	0.0054	0.0053
	0.6	0.0090	0.0050	0.0042
β_{22} (i=8)	0.2	0.0635	0.0360	0.0358
	0.4	0.0582	0.0315	0.0309
	0.6	0.0523	0.0290	0.0242
k_1 (i=9)	0.2	0.0316	0.0047	0.0045
	0.4	0.0283	0.0039	0.0038
	0.6	0.0221	0.0120	0.0027
k_2 (i=10)	0.2	0	0.0225	0.0222
	0.4	0	0.0196	0.0191
	0.6	0	0.0263	0.0150
k_3(i=11)	0.2	7.13e-04	0.0013	0.0024
	0.4	0.0026	0.0042	0.0078
	0.6	0.0044	0.0167	0.0338

plate resting on Pasternak nonlinear elastic foundations. The combined effects of all input random variables are dominant for Pasternak nonlinear elastic foundations. When the plate is CSCS supported and resting on Winkler or Pasternak nonlinear elastic foundations there is decrease in mean values and increase in COV. The influence of Clamped support with the plate resting on Pasternak nonlinear elastic foundations is dominant.

Table 6(a) shows the effects of no of layers, nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10] on the dimensionlised mean (T^*_{crnl} = T_{crnl} e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load, plate thickness ratio (a/h) =40, of angle ply laminated square composite plate subjected to simple boundary support S2, uniform constant temperature and

moisture, in-plane uni-axial compression, rise in temperature and moisture ΔT= 325K, ΔC= 0.010, (T^*_{crl} = T_{crl}e+08) - mean dimensionalized linear solution. It is observed that for a lay-up cross ply laminated composite plate simply supported and resting on Winkler nonlinear elastic foundations in study, the increase in amplitude ratio increases the mean values and COV decreases. The mean values are further increased and COV is decreased when the plate is resting on Pasternak nonlinear elastic foundations. The combined effects of all random variables are dominant besides β_{22} effects on coefficient of variations. With the increase of number of layers, mean values further increases marginally and COV decrease. The mean values and COV are higher compared to cross ply laminated composite plate with similar foundation parameters.

Table 6 (b) shows the effects of no of layers, nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10] on the mean (T^*_{crnl} = T_{crnl} e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load, plate thickness ratio (a/h) =40, of angle ply laminated square composite plate subjected to simple boundary support S2, uniform constant temperature and moisture, in-plane uni-axial compression, rise in temperature and moisture ΔT= 325K, ΔC= 0.010, (T^*_{crl} = T_{crl}e+08) - mean linear solution. It is noticed that for angle ply laminated composite plate simply supported and resting on Winkler nonlinear elastic foundations in study, the increase in amplitude ratio increase the mean values and COV decrease. The expected mean values are further increased and coefficient of variations is decreased when the plate is resting on Pasternak nonlinear elastic foundations. The combined effects of all random input variables are dominant besides β_{22} effects on coefficient of variations. With the increase of number of layers mean values further increase marginally.

TABLE 3(A)

BC's	a/h	W_{max}/h	(T_{crnl})	COV, λ_{crnl}			
				bi			
				(i=1..8)	(i=6,7)	(i=8)	(i=9)
SSSS(S1)	40	0.2	0.0112	0.2040	0.0664	0.0922	0.0353
		0.4	0.0124	0.1895	0.0600	0.0833	0.0326
		0.6	0.0141	0.1739	0.0526	0.0730	0.0295
	60	0.2	0.0034	0.3014	0.1474	0.2047	0.0354
		0.4	0.0037	0.2755	0.1331	0.1848	0.0324
		0.6	0.0042	0.2462	0.1165	0.1618	0.0290
	80	0.2	0.0014	0.4694	0.2576	0.3578	0.0349
		0.4	0.0016	0.4271	0.2332	0.3239	0.0320
		0.6	0.0018	0.3777	0.2046	0.2841	0.0286
CCCC	40	0.2	0.0356	0.2481	0.0208	0.0290	0.0403
		0.4	0.0376	0.2407	0.0197	0.0274	0.0392
		0.6	0.0407	0.2319	0.0182	0.0253	0.0379
	60	0.2	0.0113	0.2405	0.0439	0.0610	0.0412
		0.4	0.0119	0.2323	0.0415	0.0577	0.0396
		0.6	0.0130	0.2218	0.0382	0.0530	0.0374
	80	0.2	0.0049	0.2524	0.0758	0.1052	0.0419
		0.4	0.0052	0.2424	0.0715	0.0994	0.0400
		0.6	0.0056	0.2290	0.0656	0.0911	0.0373

TABLE 3(B)

Foundation parameters	W_{max}/h	a/h=10					a/h=30				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9,11)		(i=1..11)	(i=6,7)	(i=8)	(i=9,11)
(k_1=100, k_2=00 ,k_3=100)	0.2	0.7673	0.1100	0.0027	0.0156	0.0087	0.1170	0.1826	0.0179	0.1024	0.0238
	0.4	0.8318	0.2065	0.0025	0.0144	0.0304	0.1464	0.1514	0.0143	0.0818	0.0088
	0.6	0.8689	0.1609	0.0024	0.0138	0.0211	0.1690	0.1508	0.0124	0.0709	0.0110
	T_{crl}	(0.6684)					(0.0835)				
(k_1=100, k_2=10, k_3=100)	0.2	1.0354	0.1153	0.0020	0.0231	0.0299	0.1583	0.1899	0.0132	0.0757	0.0233
	0.4	1.2473	0.1157	0.0021	0.0119	0.0299	0.1819	0.1763	0.0115	0.0658	0.0196
	0.6	1.1632	0.1088	0.0017	0.0103	0.0284	0.2069	0.1308	0.0101	0.0579	0.0183
	T_{crl}	(0.9855)					(0.1412)				
(k_1=100, k_2=10, k_3=200)	0.2	1.0664	0.1128	0.0020	0.0112	0.0288	0.1597	0.1436	0.0131	0.0750	0.0227
	0.4	1.1353	0.1166	0.0018	0.0105	0.0308	0.1859	0.1370	0.0112	0.0644	0.0202
	0.6	1.0489	0.1199	0.0020	0.0114	0.0319	0.2134	0.1374	0.0098	0.0561	0.0216
	T_{crl}	(0.9855)					(0.1359)				

TABLE 4

Foundation parameters	W_{max}/h	a/b =1					a/b =2				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9..11)		(i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 k_3=100)	0.2	0.1329	0.1503	0.0157	0.0901	0.0051	0.0463	0.2804	0.0451	0.2587	0.0022
	0.4	0.1555	0.1385	0.0134	0.0770	0.0040	0.0684	0.2012	0.0305	0.1749	0.0019
	0.6	0.1775	0.1355	0.0118	0.0674	0.0055	0.0835	0.1766	0.0250	0.1434	0.0019
	T_{crl}	(0.1107)					(0.0339)				
(k_1=100, k_2=10, k_3=100)	0.2	0.1521	0.1584	0.0137	0.0787	0.0138	0.0512	0.2637	0.0408	0.2337	0.0099
	0.4	0.1680	0.1499	0.0124	0.0713	0.0128	0.0732	0.1000	0.0285	0.1635	0.0067
	0.6	0.1891	0.1411	0.0110	0.0633	0.0115	0.0881	0.1749	0.0237	0.1359	0.0055
	T_{crl}	(0.1413)					(0.0389)				
(k_1=100, k_2=10, k_3=200)	0.2	0.1524	0.1589	0.0137	0.0785	0.0138	0.0514	0.2636	0.0406	0.2329	0.0099
	0.4	0.1692	0.1520	0.0123	0.0708	0.0129	0.0737	0.1972	0.0283	0.1624	0.0071
	0.6	0.1917	0.1453	0.0109	0.0625	0.0122	0.0888	0.1760	0.0235	0.1348	0.0061
	T_{crl}	(0.1413)					(0.0389)				

TABLE 5 (A)

Foundation parameters	W_{max}/h	SSSS (S1)					SSSS (S2)				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9..11)		(i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 ,k_3=100)	0.2	0.2794	0.1280	0.0075	0.0429	0.0044	0.2744	0.1302	0.0076	0.0436	0.0037
	0.4	0.3221	0.1236	0.0065	0.0372	0.0042	0.3176	0.1255	0.0066	0.0377	0.0040
	0.6	0.3438	0.1359	0.0061	0.0348	0.0101	0.4141	0.1507	0.0050	0.0289	0.0139
	T_{crl}	(0.2387)					(0.2381)				
(k_1=100, k_2=10, k_3=100)	0.2	0.3369	0.1580	0.0062	0.0355	0.0199	0.4890	0.1313	0.0043	0.0245	0.0126
	0.4	0.3471	0.1402	0.0060	0.0345	0.0146	0.3915	0.1464	0.0053	0.0306	0.0163
	0.6	0.4023	0.1373	0.0052	0.0298	0.0141	0.3376	0.1476	0.0062	0.0355	0.0169
	T_{crl}	(0.3033)					(0.2982)				
(k_1=100, k_2=10, k_3=200)	0.2	0.3490	0.1384	0.0060	0.0343	0.0147	0.4368	0.1245	0.0048	0.0274	0.1115
	0.4	0.4221	0.1328	0.0049	0.0284	0.0124	0.3267	0.1543	0.0064	0.0366	0.0183
	0.6	0.4081	0.1397	0.0051	0 .0293	0.0119	0.3281	0.1520	0.0064	0.0365	0.0158
	T_{crl}	(0.3033)					(0.2982)				

TABLE 5 (B)

Foundation parameters	W_{max}/h	CCCC					CSCS				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9..11)		(i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 ,k_3=100)	0.2	0.4227	0.1285	0.0049	0.0283	0.0017	0.3632	0.1331	0.0058	0.0330	0.0051
	0.4	0.5262	0.1331 0.1356	0.0040	0.0228	0.0038	0.3256	0.1359	0.0064	0.0368	0.0049
	0.6	0.5130		0.0041	0.0233	0.0055	0.4058	0.1325	0.0051	0.0295	0.0054
	T_{crl}	(0.3758)					(0.2878)				
(k_1=100, k_2=10, k_3=100)	0.2	0.4691	0.1354	0.0045 0.0039	0.0511	0.0096	0.4114	0.4114	0.0051	0.0291	0.0142
	0.4	0.5375	0.1324	0.0042	0.0223	0.0089	0.3753	0.1480	0.0056	0.1319	0.0140
	0.6	0.4938	0.1361		0.0242	0.0098	0.4591	0.1427	0.0045	0.0261	0.0127
	T_{crl}	(0.4306)					(0.3433)				
(k_1=100, k_2=10, k_3=200)	0.2	0.5521	0.1359	0.0038	0.0179	0.0095	0.4124	0.1467	0.0051	0.0290	0.0140
	0.4	0.5466	0.1300	0.0038	0.0219	0.0086	0.3777	0.1489	0.0055	0.0317	0.0140
	0.6	0.5338	0.1313	0.0039	0.0224	0.0096	0.4064	0.1356	0.0051	0.0295	0.0115
	T_{crl}	(0.4306)					(0.3433)				

Table 7(a) shows the effects of environmental conditions, nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10] on the dimensionlised mean (T^*_{crnl} =T_{crnl}e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load of angle ply [±45^0]$_{2T}$ laminated square composite plates with SSSS (S2) boundary conditions subjected to in-plane uni-axial compression , plate thickness ratio (a/h)=20. (T^*_{crl} = T_{crl}e+08) - mean dimensionalized linear solution. It is seen that for given environmental conditions the increase in amplitude ratios increases the mean values and coefficient of variations decreases. The combined effects of all random input variables increase the coefficient of variations when plate is resting on Winkler nonlinear elastic foundations. On change of foundation parameters to Pasternak these values further increase and coefficient of variations decreases. With the increase of moisture contents keeping temperature constant the effects of amplitude ratios decrease the mean values for Winkler nonlinear elastic foundations, however these values are higher for Pasternak nonlinear elastic foundations.

Table 7(b) shows the effects of environmental conditions, nonlinear foundation parameters and amplitude ratios (W_{max}/h) with random input variables b_i,[{(i =1 .. 11), (6, 7),(8) and (9..11)} = 0.10] on the dimensionlised mean (T^*_{crnl} =T_{crnl}e+08) N/m and dimensionless coefficient of variation (λ_{crnl}) of hygrothermal post buckling load of angle ply [±45^0]$_{2T}$ laminated square composite plates with SSSS (S2) boundary conditions subjected to in-plane uni-axial compression , plate thickness ratio (a/h)=20. (T^*_{crl} = T_{crl}e+08) - mean dimensionalized linear solution. It is seen that for given environmental conditions the increase in amplitude ratios increases the mean values and decreases the COV. The combined effects of all random input variables increase the COV, when plate is resting on Winkler nonlinear elastic foundations. On change of foundation parameters to Pasternak these values further increase and COV decreases. With the increase of moisture contents keeping temperature constant the effects of amplitude ratios decreases the mean values for Winkler nonlinear elastic foundations, however these values are higher for Pasternak nonlinear elastic foundations. Increase in environmental conditions drastically decreases the mean values and coefficient of variations.

TABLE 6 (A)

Foundation parameters	W_{max}/h	[0^0/90^0/]$_{2T}$ (T_{crnl})	COV, λ_{crnl} bi (i=1..11)	(i=6,7)	(i=8)	(i=9..11)	[0^0/90^0/]$_{4T}$ (T_{crnl})	COV, λ_{crnl} bi (i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 ,k_3=100)	0.2	0.0672	0.2137	0.0311	0.1781	0.0185	0.0930	0.1655	0.0225	0.1288	0.0088
	0.4	0.0859	0.1781	0.0243	0.1393	0.0089	0.1218	0.1465	0.0171	0.0983	0.0136
	0.6	0.1000	0.1642	0.0209	0.1198	0.0109	0.1487	0.1437	0.0140	0.0805	0.0211
	T_{crl}	(0.0477)					(0.0511)				
(k_1=100, k_2=10, k_3=100)	0.2	0.0917	0.1815	0.0228	0.1306	0.0241	0.1131	0.1558	0.0185	0.1058	0.0182
	0.4	0.1060	0.1642	0.0197	0.1129	0.0191	0.1422	0.1438	0.0147	0.0842	0.0178
	0.6	0.1204	0.1554	0.0174	0.0995	0.0181	0.1701	0.1431	0.0123	0.0704	0.0220
	T_{crl}	(0.0772)					(0.0806)				
(k_1=100, k_2=10, k_3=200)	0.2	0.0926	0.1808	0.0225	0.1293	0.0231	0.1156	0.1571	0.0181	0.1036	0.0188
	0.4	0.1652	0.1000	0.0193	0.1105	0.0198	0.1498	0.1501	0.0139	0.0799	0.0236
	0.6	0.1250	0.1585	0.0167	0.0958	0.0211	0.1930	0.1439	0.0108	0.0620	0.0265
	T_{crl}	(0.0772)					(0.0806)				

TABLE 6 (B)

Foundation parameters	W_{max}/h	$[\pm 45^0]_{2T}$					$[\pm 45^0]_{4T}$				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9..11)		(i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 ,k_3=100)	0.2	0.0772	0.1978	0.0270	0.1550	0.0066	0.0996	0.1617	0.0210	0.1202	0.0038
	0.4	0.0912	0.1744	0.0229	0.1313	0.0039	0.1273	0.1474	0.0164	0.0940	0.0073
	0.6	0.1051	0.1626	0.0199	0.1140	0.0054	0.1499	0.1473	0.0139	0.0799	0.0121
	T_{crl}	(0.0635)					(0.0675)				
(k_1=100, k_2=10, k_3=100)	0.2	0.0894	0.1907	0.0234	0.1339	0.0128	0.1106	0.1633	0.0189	0.1082	0.0105
	0.4	0.1009	0.1752	0.0207	0.1187	0.0115	0.1411	0.1490	0.0148	0.0849	0.0099
	0.6	0.1160	0.1620	0.0180	0.1032	0.0104	0.1662	0.1491	0.0126	0.0720	0.0127
	T_{crl}	(0.0809)					(0.0849)				
(k_1=100, k_2=10, k_3=200)	0.2	0.0897	0.1911	0.0233	0.1335	0.0128	0.1117	0.1654	0.0187	0.1072	0.0108
	0.4	0.1017	0.1770	0.0205	0.1177	0.0118	0.1450	0.1564	0.0144	0.0826	0.0132
	0.6	0.1180	0.1659	0.0177	0.1015	0.0117	0.1747	0.1625	0.0120	0.0686	0.0196
	T_{crl}	(0.0809)					(0.0849)				

TABLE 7 (A)

Foundation parameters	W_{max}/h	ΔT= 300K ΔC=0.001					ΔT= 300K ΔC=0.0025				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9..11)		(i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 ,k_3=100)	0.2	2.3511	0.2502	0.0081	0.0045	0.0105	1.0476	0.2453	0.0073	0.0103	0.0103
	0.4	2.5527	0.2512	0.0074	0.0041	0.0093	1.1319	0.2458	0.0068	0.0095	0.0121
	0.6	2.7029	0.2551	0.0070	0.0039	0.0095	1.1800	0.2772	0.0065	0.0091	0.0059
	T_{crl}	(2.1164)					(0.8604)				
(k_1=100, k_2=10, k_3=100)	0.2	3.2970	0.2367	0.0058	0.0032	0.0176	1.1909	0.2372	0.0064	0.0090	0.0200
	0.4	3.2297	0.2369	0.0059	0.0030	0.0171	1.7236	0.2249	0.0044	0.0062	0.0098
	0.6	3.1101	0.2517	0.0070	0.0034	0.0180	1.2950	0.2462	0.0059	0.0083	0.0143
	T_{crl}	(2.6525)					(1.0782)				
(k_1=100, k_2=10, k_3=200)	0.2	2.8909	0.2397	0.0066	0.0040	0.0186	1.1520	0.2587	0.0067	0.0093	0.0156
	0.4	3.8574	0.2386	0.0049	0.0027	0.0125	1.6026	0.2526	0.0048	0.0067	0.0104
	0.6	3.1339	0.2385	0.0061	0.0036	0.0176	1.3021	0.2468	0.0059	0.0083	0.0143
	T_{crl}	(2.6525)					(1.0782)				

TABLE 7 (B)

Foundation parameters	W_{max}/h	ΔT= 325K ΔC=0.0125					ΔT= 325K ΔC=0.015				
		(T_{crnl})	COV, λ_{crnl}				(T_{crnl})	COV, λ_{crnl}			
			bi					bi			
			(i=1..11)	(i=6,7)	(i=8)	(i=9..11)		(i=1..11)	(i=6,7)	(i=8)	(i=9..11)
(k_1=100, k_2=00 ,k_3=100)	0.2	0.2197	0.1342	0.0076	0.0545	0.0037	0.1832	0.1390	0.0076	0.0654	0.0037
	0.4	0.2543	0.1286	0.0066	0.0471	0.0040	0.2120	0.1324	0.0066	0.0565	0.0040
	0.6	0.2855	0.1303	0.0059	0.0420	0.0058	0.2381	0.1332	0.0059	0.0503	0.0058
	T_{crl}	(0.1906)					(0.1589)				
(k_1=100, k_2=10, k_3=100)	0.2	0.2830	0.1548	0.0059	0.0423	0.0182	0.2359	0.1568	0.0059	0.0508	0.0180
	0.4	0.1490	0.1000	0.0065	0.0463	0.0155	0.2842	0.1362	0.0049	0.0422	0.0116
	0.6	0.2967	0.1594	0.0056	0.0404	0.0175	0.2159	0.1529	0.0065	0.0555	0.0155
	T_{crl}	(0.2387)					(0.1990)				
(k_1=100, k_2=10, k_3=200)	0.2	0.2856	0.1558	0.0059	0.0419	0.0176	0.2380	0.1573	0.0059	0.0503	0.0173
	0.4	0.2704	0.1451	0.0062	0.0443	0.0146	0.2882	0.1357	0.0048	0.0416	0.0113
	0.6	0.3406	0.1393	0.0049	0.0352	0.0126	0.4070	0.1130	0.0034	0.0294	0.0104
	T_{crl}	(0.2387)					(0.1990)				

Conclusions

The stochastic DISFEM procedure outlined in the present study has been used to obtain the mean and COV of the hygrothermal postbuckling load of the laminated composite plates without foundations and resting on nonlinear elastic foundations, subjected to uniform temperature and moisture rise with random system parameters. The following conclusion can be drawn from this limited study:

1. For various input random variables, amplitude ratios of cross ply laminate, the increase in temperature and moisture concentration effect significantly as the hygrothermal postbuckling load decreases drastically specially for clamped support conditions.
2. The random system properties, E_{11}, E_{22}, G_{12}, α_{22}, h and foundation parameters have dominant effect on the coefficient of variation of the hygrothermal post buckling load as compared to other system properties subjected to both temperature and moisture dependent hygrothermo-elastic properties. The strict control of these random parameters is therefore, required if high reliability of the laminated composite is desired.
3. When the combined effects of aspect ratio and amplitude ratio along with random variables in hygrothermal environment are studied, then the clamped support conditions are significantly affected by the hygrothermal post buckling load compared to simple support conditions. However, coefficient of variation of hygrothermal postbuckling load for composite plates subjected to clamped support condition is lower as compared to simply support conditions for both without foundation and with nonlinear elastic foundations.
4. The mean hygrothermal post buckling load for the parameters considered as mentioned above is more pronounced for square composite plates compared to rectangular plates; however it is vice versa in the case of coefficient of variation of hygrothermal postbuckling load for composite plates. The effect of hygrothermal load is more significant for coefficient of variation when number of layers increase.
5. In general, the hygrothermal effects are of due importance for analyzing the structural response made of composite laminate. The negligence of

hygrothermal effects in analyzing the system behavior may leave the design unsafe for manufacturing and may prove to be a tool to avoid failure for reliable operational requirements.

REFERENCES

Agarwal, B.D., and Broutman, L.J. "Analysis and performance of fiber composites", Second edition, *John Wiley & sons*, New York, 1990.

Chen, S., Lin, Z. and Zhang, Z. "Random Variation Analysis for larger scale structures with random parameters", *Compos. Struct.* Vol. 43, pp 247-254, 1992.

Chia, C.Y. "Nonlinear analysis of plates", *McGraw-Hill*, New York, 1980.

Englested, S.P. and Reddy, J.N. "Probabilistic methods for the analysis of matrix composite", *Compos Sci Techn*, vol. 50, pp 91–107, 1994.

Flaggs, D.L. and Vinson, J.R. "Hygrothermal effects on the buckling of laminated composite plates", *Fiber Sci.Technology* , vol. 1, pp353–65, 1978.

Franklin, J.N. "Matrix theory", *Englewood cliffs*. JN Prentice Hall, 1968.

Graham, L.L. and Deodatis, G. "Response and eigenvalue analysis of stochastic finite element systems with multiple correlated material and geometric properties", *Probab. Eng. Mech* pp1611–29, 2001.

Graham, L.L. and Siragy, E.F. "Stochastic finite element analysis for elastic buckling of stiffened panels", *J Eng. Mech*, vol. 127, pp 91–7, 2001.

Jones, R.M. "Mechanics of composite materials", *McGraw-Hill*, New York, 1975.

Kleiber, M. and Hien, T.D. "The Stochastic finite element method" *Wiley*, New York, 1992.

Lal, A., Singh, B.N. and Kumar, R . "Effect of random system properties on initial buckling of composite plates resting on elastic foundation", Int. *J Struct. Stab. Dyn*, vol. 1, pp103-130, 2008.

Lal, A., Singh, B.N. and Kumar, Rajesh. "Effects of random system properties on the thermal buckling analysis of laminated composite plates", *Comput. & Struct.*, vol. 87 (17-18) , pp1119-1128, 2009.

Lee, S.Y. Chou C.J., Jang, J.L. and Lin, J.S. "Hygrothermal effects on linear and non-linear analysis of symmetric angle-ply laminated plates", *Compos Struct*, vol. 21, pp41-48, 1992.

Liu, W.K., Belytschko, T. and Mani, "A Random field finite elements", *Int J Numer. Meth Engrg*, vol. 23, pp1831-1845, 1986.

Nakagiri, H. S. Tatabatake and Tani, S. "Uncertain eigen value analysis of composite laminated plates by SFEM", *Compos Struct* , pp149–12, 1990.

Onkar, A.K., Upadhyay C.S. and Yadav, D. "Generalized buckling analysis of laminated plates with random material properties using stochastic finite elements", *Int. J Mech. Sci* , vol. 48, pp780–798, 2006.

Onkar, A.K., Upadhyay, C.S. and Yadav, D. "Stochastic finite element buckling analysis of laminated plates with circular cutout under uniaxial compression", *J Appl. Mech*, vol. 74(4), pp 798 -810, 2007.

Patel, B.P., Ganapati, M. and Makhecha, "Hygrothermal effects on the structural behaviour of thick composite laminates using higher-order theory", *Compos Struct*, vol. 56, pp 25-34, 2002.

Ramesh. Pandey, Shukla, K.K. and Anuj Jain, "Thermoelastic stability analysis of laminated composite plates", An analytical approach, *Communications in Nonlinear Science and Numerical Simulation*, vol.14(4), pp1679-1699, 2009.

Reddy, J.N. "A Simple higher order theory for laminated composite plates", *ASME J Appl Mech*, vol. 51, pp745–752, 1984.

Reddy, J.N. "Energy and Variational methods in applied mechanics", *Wiley*, New York, 1981.

Reddy, J.N. "Mechanics of laminated composite plates, theory and analysis", *CRC press*, Boca Raton Florida, 1997.

SaiRam, K.S. and Sinha, P.K. "Hygrothermal effects on the buckling of laminated composite plates", *Compos Struct* , vol. 21, pp 233- 247, 1992.

Shankara, C.A. and Iyenger, N.G.R. "A C^0 element for the free vibration analysis of laminated composite plates" *J Sound. Vib*, vol. 191 (5) , pp 721–738. 1996.

Shen, H.S. "Hygrothermal effects on the post buckling of shear deformable laminated plates", *Int. J Mech. Sci* , vol. 43, pp1259-1281, 2001.

Singh, B.N., and Yadav, D. and Iyengar, N.G.R. "Initial buckling of composite cylindrical panels with random material properties", *Compos. Struct* , vol. 53(1) , pp 55-64, 2001.

Singh, B.N., and Verma, V.K. "Hygrothermal effects on the buckling of laminated composite plates with random geometric and material properties", *J Reinf. Plast and Compos*, vol. 28(4), pp 409-427, 2009.

Singh, B.N., Iyengar N.G.R., and Yadav, D. "A C^0 Finite element investigation for buckling analysis of composite plates with random material properties", *Struct Engrg and Mech*, vol. 113(1), pp 53-74, 2002.

Singh, B.N., Iyengar, N.G.R. and Yadav, D. "Effects of random material properties on buckling of composite plates", *ASCE J Eng. Mech*, vol. 127(9), pp 873-879, 2001.

Singh, B.N. Lal, A. and Kumar, R. "Post buckling of laminated composite plate on elastic foundation with random system properties", *Communi. Nonlr. Sci Numer. Simul*, vol. 14, pp284-300, 2009.

Singh, B.N. Yadav, D.and Iyenger, N.G.R. "Stability analysis of laminated composite cylindrical panels with uncertain material properties", *Compos Structm*, vol. 54, pp 17-26, 2001.

Verma, V.K., and Singh, B.N. "Thermal buckling of laminated composite plates with random geometric and material properties", Int *J Struct. Stab. Dyn*., vol. 9(2) , pp187-211, 2009.

Whitney, J.M. and Ashton, J.E., "Effect of environment on the elastic response of layered composite plates", *AIAA*, vol. 9, pp1708–13, 1971.

Yamin, Z., Chen, S. and Lue, Q. "Stochastic perturbation finite elements", *Comp and Struct*, vol. 59 (3), pp 425- 429, 1996.